Suspension Bridges and the Parabolic Curve

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
The student will examine the phenomenon of suspension bridges and see how the parabolic curve strengthens the construction. The student will then diagram their own bridge given a scenario and find key points using a quadratic equation.

II. UNIT AUTHOR:
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III. COURSE:
Algebra II

IV. CONTENT STRAND:
Equations and Inequalities, Functions

V. OBJECTIVES:
The student will be able to:

- investigate and analyze quadratic functions both algebraically and graphically
- make connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

VI. REFERENCE/RESOURCE MATERIALS:
Graphing calculator, handout (included), and computer access.

VII. PRIMARY ASSESSMENT STRATEGIES:
The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. The assessment for the student activity is attached.

VIII. EVALUATION CRITERIA:
Assessment list for Student Activity, corresponding rubric.

IX. INSTRUCTIONAL TIME:
The activity is designed for 2 blocks (90 minutes each).

Suspension Bridges and the Parabolic Curve

Strand
Equations and Inequalities, Functions

Mathematical Objective(s)
The student will be able to:

- investigate and analyze quadratic functions both algebraically and graphically
- make connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

Related SOL

AII.7 (analyzing quadratic functions graphically and algebraically)

NCTM Standards

- use symbolic algebra to represent and explain mathematical relationships
- build new mathematical knowledge through problem solving
- recognize and apply mathematics in contexts outside of mathematics

Materials/Resources
- Graphing Calculators
- Graph Paper
- Computer Lab or Computer Access

Assumption of Prior Knowledge

Students should have basic knowledge of quadratic equations and the nature of a parabola to include the vertex form of a quadratic equation.

$$y = a(x-h)^2 + k, where\ (h,k) is\ the\ vertex\ of\ the\ parabola$$

Students should have a basic understanding of the symmetry of quadratic functions.

Introduction: Setting Up the Mathematical Task

In this activity, you will investigate the wonder of suspension bridges and the science behind their construction. You will learn about tension and compression forces and how a parabolic curve between supports on a bridge helps achieve maximum strength. You will also design your own bridge and plot the points on a coordinate plane paying special attention to the coordinates of the intersection of the cable and hangers.

Student Exploration

Optional: Review the concept of symmetry of parabolas and the vertex form of a quadratic equation.

As an entire class or individually in a computer lab, have students view the following video:

http://www.youtube.com/watch?v=rbrhwTvrxHk

After viewing the short video direct students to the following website and have them read the simple explanation of suspension bridges:

http://www.carondelet.pvt.k12.ca.us/Family/Math/03210/page4.htm

Explain to students that in this activity, they will be diagraming a suspension bridge with certain characteristics. They will be presented with a scenario and a list of bridge requirements and they will draw their bridge on graph paper. They will then be asked to answer a series of questions about their bridge. This activity will take 2 class periods.

Teacher actions:

It is recommended that the teacher lead a discussion about the video and the reading from the website. Be sure the students understand the following characteristics of a suspension bridge:

Supports: The towers of the bridge.

Span: Distance between two towers on suspension bridge.

Approaches: The roads leading up to the bridge.

Cable: Runs from tower to tower and connects to hangers.

Hanger: Connects the cable to the roadway of the bridge.

The teacher should also move throughout the room to help students who may be struggling with certain aspects of the task. In particular, the estimation for the amount of cable for the parabolic portion of the bridge can be done using Pythagorean Theorem. (See benchmark)

Give the students the following scenario (included on the handout):

A bridge has failed just as in the video. As an up and coming engineer, you have been asked to help with the design of a new bridge. In the preliminary stages, you are mostly concerned with the area between the two pillars of the bridge as that is where the problem with the last bridge seems to have originated. The bridge must span a 2000 meter section of a bay. For the purposes of your early design you are to assume the following:

- The end of the approach from the west side of the bay will represent point (0, 40) on your coordinate plane.
- There is a 40 m drop from the end of the approaches to the body of water below.

Note: The teacher may alter the handout and not give the information above depending on the level of your students. Some students may not need this starting point on their graph.

Student actions: (this may be done in small cooperative learning groups or individually)

Using the handout and the included coordinate plane, the student will:

- Draw a plan for a bridge on the coordinate plane that includes two square supports, at least one cable from support to support on each side of the road, and as many hangers as the student feels necessary to attach the road to the cables.
- Answer the following questions:

1. How tall and wide are your supports?
2. What are the coordinates of the top your supports?
3. What is the length of your shortest hanger?
4. What are the coordinates of the vertex of the cable that extends from support to support?
5. What is the equation of your parabolic cable?
6. How many additional hangers are you using on your bridge between the two supports?
7. What are the intersections of your other hangers with the parabolic cable?

8. How much steel cable will you need for all your hangers on both sides of the road?
9. Try to estimate how much cable you will need for the parabolic cable that stretches between the two supports.

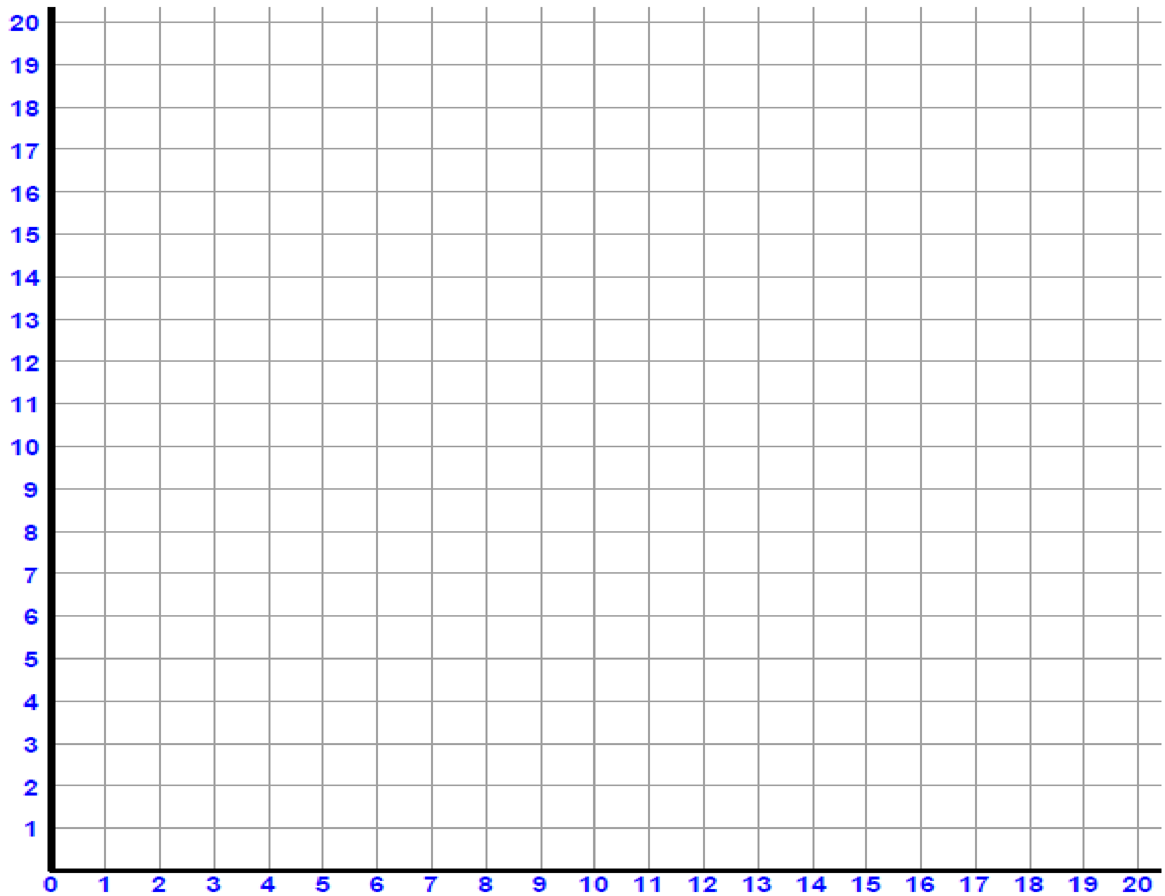
Note: Depending on the level of your students, it may not be necessary to step them through this process with these questions. If you wish to make the activity much more open-ended, you could just give the students the very broad task of "Calculating the amount of cable they will need for all parabolic cables and hangers using their knowledge of quadratic functions." Have them show work and explain their amounts using points on the coordinate plane.

Suspension Bridge worksheet:

A bridge has failed just as in the video. As an up and coming engineer, you have been asked to help with the design of a new bridge. In the preliminary stages, you are mostly concerned with the **area between the two pillars of the bridge** as that is where the problem with the last bridge seems to have originated. The bridge must span a 2000 meter section of a bay. For the purposes of your early design you are to assume the following:

- The end of the approach from the west side of the bay will represent point (0, 40) on your coordinate plane.
- There is a 40 m drop from the end of the approaches to the body of water below.
- Let each mark on the x-axis represent 100m and each mark on the y-axis represent 10m.

Draw a plan for a bridge on the coordinate plane that includes two square supports, at least one cable from support to support on each side of the road, and as many hangers as you feel are necessary to attach the road to the cables. Remember, you are dealing with the area between the supports. Another engineering firm is being brought in to address the anchors of the bridge. Label the axes according to the instructions.



Answer the following questions:

1. How tall and wide are your supports?

2. What are the coordinates of the top your supports?

3. What is the length of your shortest hanger?

4. What are the coordinates of the vertex of the cable that extends from support to support?

5. What is the equation of your parabolic cable? Show the process you used to determine the equation.

6. How many additional hangers are you using on your bridge between the two supports? Why did you use this additional number of hangers?

7. What are the intersections of your other hangers with the parabolic cable?

8. How much steel cable will you need for all your hangers on both sides of the road?

9. Try to estimate how much cable you will need for the parabolic cable that stretches between the two supports.

Assessment List and Benchmarks

Assessment List for Student Activity: Suspension Bridge Design

Number	Element	Point Value	Self	Teacher
1	Diagram of the bridge	2		
2	Supports	2		
3	Hangers	2		
4	Vertex	2		
5	Coordinates of Supports	2		
6	Vertex form of quadratic equation	2		
7	Coordinates of hangers	2		
8	Cable needed for hangers	2		
9	Cable needed for parabolic cable	2		
	Total	18		

Rubric for Activity 1: Data Collection

Number	Element	0	1	2
1	Diagram of bridge	No elements of the diagram are correct.	The diagram has at least one axis correctly labeled and scaled and the general shape of a parabola is observed.	The diagram has both axes correctly labeled and scaled and the general shape of a parabola is observed.
2	Supports	No supports drawn.	At least one support.	Both supports are present and positioned correctly.
3	Hangers	There is no representation of the hangers.	There is representation of hangers but in inappropriate positions.	The hangers are all present and in appropriate positions.
4	Vertex	The vertex is not marked or labeled.	The vertex is marked with at least one correct coordinate.	The vertex is marked and both coordinates are correct.
5	Coordinates of supports	The top of the supports are not marked or labeled.	The top of the supports are marked with at least one correct coordinate.	The top of the supports are marked with all correct coordinates.
6	Vertex form of quadratic equation	There is no equation derived.	An equation is derived but is not correct or in the correct vertex form.	An equation is derived and is in the correct vertex form.
7	Coordinates of Hangers	There are no coordinates for the other hangers.	Between 1 and 4 sets of coordinates are correct for the hangers.	5 or more sets of coordinates for the hangers are correct.
8	Cable needed for hangers	No answer or there is no evidence that the student is close to the correct total.	An attempt is made but the student is not within 20 meters of the correct answer.	The student is within 20 meters of the correct answer.
9	Cable needed for parabolic cable	No answer or there is no evidence that the student is close to a correct total.	An attempt is made but the student is not within 20 meters of the correct answer based on their self-determined criteria.	The student is within 20 meters of the correct answer based on their self-determined criteria.

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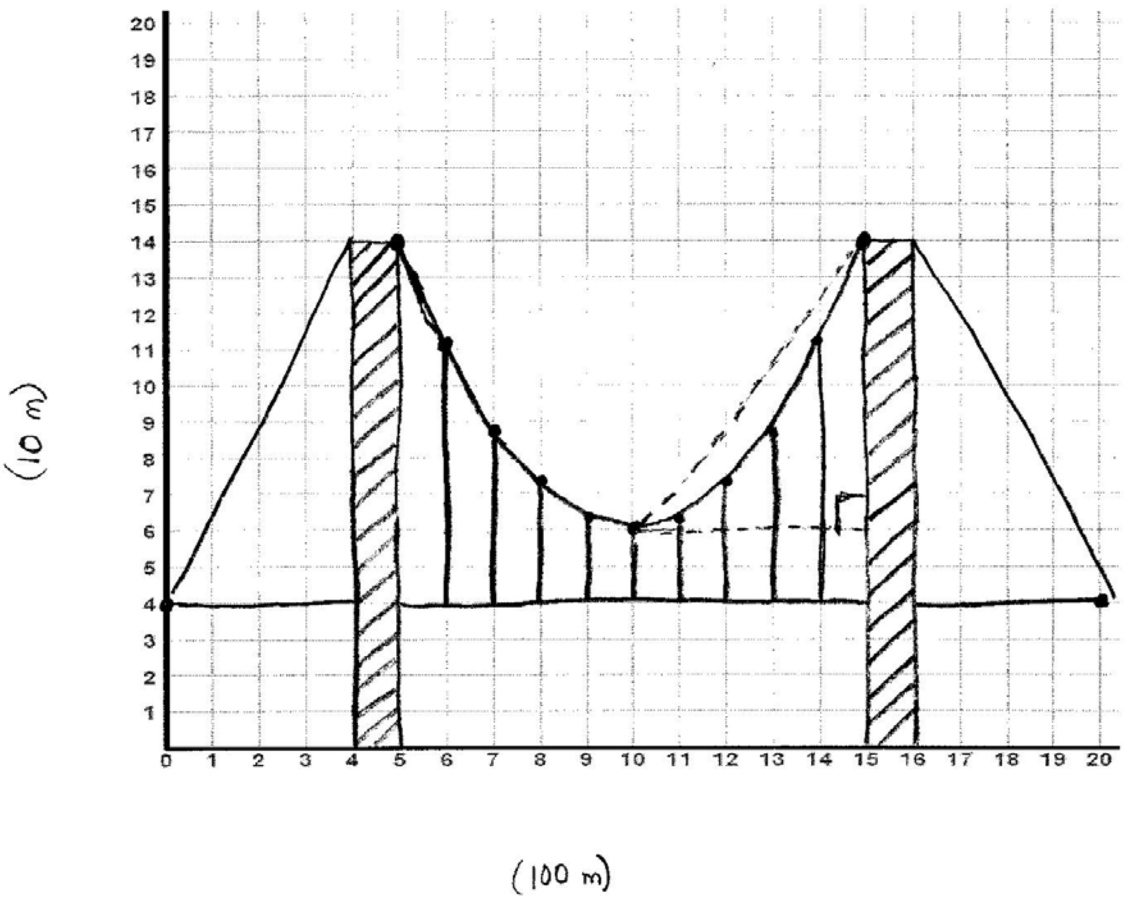
Draw a plan for a bridge on the coordinate plane that includes two square supports, at least one cable from support to support on each side of the road, and as many hangers as you feel are necessary to attach the road to the cables. Remember, you are dealing with the area between the supports. Another engineering firm is being brought in to address the anchors of the bride. Label the axes according to the instructions.



Answer the following questions:

1. How tall and wide are your supports? Two 10m × 10m supports. They are 140 m tall.

2. What are the coordinates of the top your supports?

(500, 140) and (1500, 140)

3. What is the length of your shortest hanger?

20 meters

4. What are the coordinates of the vertex of the cable that extends from support to support?

(1000, 60)

5. What is the equation of your parabolic cable?

$$y = a(x-h)^2 + K$$

$$y = a(x-1000)^2 + 60$$

Substitute: $140 = a(500-1000)^2 + 60$

$$140 = 250000a + 60$$

$$80 = 250{,}000a$$

$$3.2 \times 10^{-4} = a$$

$$\boxed{y = .00032(x-1000)^2 + 60}$$

6. How many additional hangers are you using on your bridge between the two supports?

8 hangers on each side of the road in addition to the Shortest hanger.

7. What are the intersections of your other hangers with the parabolic cable?

using TI-83 table of values for my parabola

x	y
600	111.2
700	88.8
800	72.8
900	63.2
1100	63.2
1200	72.8
1300	88.8
1400	111.2

8. How much steel cable will you need for all your hangers on both sides of the road?

one side

shortest hanger: 20 m

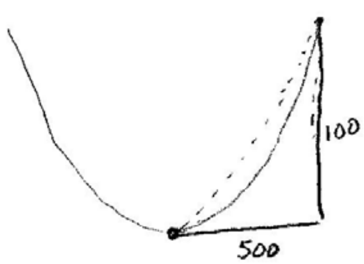
$2(111.2-40)+2(88.8-40)+2(72.8-40)+2(63.2-40)+20=372$ m

372 m × 2 sides

744 m for the hangers.

9. Try to estimate how much cable you will need for the parabolic cable that stretches between the two supports.

Using Pythagorean theorem,



$c^2 = 500^2 + 100^2$

$c^2 = 260{,}000$

$c = 510$ m

510 m × 4 segments: 2 on each side

2040 m